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Spherically Symmetric Cosmic Strings in Bimetric Theory

P.K. Sahoo

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Abstract Spherically symmetric space-time is considered in bimetric theory of gravitation formulated by Rosen (Gen. Relativ. Gravit. 4:435, 1973) in the presence of perfect fluid, massive scalar field and cosmic string. It is shown that either macro cosmological model represented by perfect fluid or cosmic string do not exist and only a vacuum model can be constructed whereas in case of a micro cosmological model represented by a scalar field exists and the model is obtained.

Keywords Bimetric theory · Perfect fluid · Scalar meson field · Cosmic string

1 Introduction

Several new theories of gravitation have been formulated which are considered to be alternatives to Einstein's theory of gravitation. The most important among them are scalar tensor theories of gravitation proposed by Brans and Dicke [1], Nordtvedt [13], Ross [22], Dunn [2] and Rosen's [20] bimetric theory of gravitation. Bimetric theory is based on a simple form of Lagrangian and has a simpler mathematical structure than that of general relativity. In this theory, at each point of space time, there exist two metric tensors: a Riemannian metric tensor g_{ij} and the background flat space time metric tensor γ_{ij}. The tensor g_{ij} describes the geometry of the curved space time and the gravitational fields while γ_{ij} refers to inertial forces. Moreover, bimetric theory also satisfies the covariance and equivalence principles, which are the foundations of general relativity. The theory also agrees with present day observational facts pertaining to general relativity.

The field equations of bimetric theory of gravitation derived from variational principle are

$$N^i_j - \frac{1}{2}N\delta^i_j = -8\pi k T^i_j \tag{1}$$

P.K. Sahoo (✉)
Department of Applied Mathematics, Birla Institute of Technology, Mesra, Ranchi 835215, India
e-mail: sahoomaku@rediffmail.com

with $N = N_i^i$
where

$$N_j^i = \frac{1}{2}\gamma^{ab}\left(g^{hi} g_{hj|a}\right)_{|b}$$

and

$$k = \left(\frac{g}{\gamma}\right)^{\frac{1}{2}}$$

together with

$$g = \text{determinant of } g_{ij} \text{ and}$$

$$\gamma = \text{determinant of } \gamma_{ij}$$

Here a vertical bar (1) stands for covariant differentiation with respect to γ_{ij} and T_j^i is the energy momentum tensor of matter fields.

Rosen [20, 21], Yilmaz [27], Karade and Dhoble [7], Karade [6] and Israelit [3–5] are some of the authors who have studied several aspects of bimetric theory of gravitation. In particular, Reddy and Venkateswarlu [14], Reddy and Venkateswar Rao [15], Mohanty and Sahoo [10, 11], Mohanty et al. [12] and Sahoo [24] have established several cosmological models in bimetric theory when the source of gravitation is governed by either perfect fluid or mesonic perfect fluid. Recently Reddy [16, 17] has presented a string cosmological model in Brans and Dicke [1] and Saez and Ballester [23] scalar tensor theories of gravitation.

In this paper, we consider the spherical symmetric cosmological model in Rosen's bimetric theory of gravitation. It is shown that these models do not exist when the source of gravitation is either perfect fluid or massive strings whereas the corresponding model can be obtained in case of scalar meson field.

2 Perfect Fluid

The energy momentum tensor for perfect fluid distribution is given by

$$T_{ij} = (\rho + p)\, U_i U_j - p g_{ij} \tag{2}$$

together with

$$U^i U_i = 1$$

where U^i is the four velocity vector of the fluid and p and ρ are the proper pressure and energy density respectively.

We consider the spherically symmetric metric in the form

$$ds^2 = dt^2 - e^{\alpha}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{3}$$

where α is function of cosmic time t only.

The background flat metric corresponding to (3) is

$$d\sigma^2 = dt^2 - \left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{4}$$

By the use of co-moving coordinate system, the field equations (1) for the metrics (3) and (4) corresponding to the energy momentum tensor (2) can be written as

$$\alpha_{44} = -32\pi\kappa p \tag{5}$$

$$3\alpha_{44} = 32\pi\kappa\rho \tag{6}$$

Here afterwards the suffix 4 after a field variable represents ordinary differentiation with respect to t.

From (5) and (6) it is a simple matter to see that

$$3p + \rho = 0 \tag{7}$$

In view of reality conditions $p \succ 0$, $\rho \succ 0$, (7) implies that

$$p = 0 \quad \text{and} \quad \rho = 0 \tag{8}$$

which, immediately, means that perfect fluid spherically symmetric cosmological models do not exist in bimetric theory of gravitation.

Hence, spherically symmetric vacuum model in bimetric theory can be expressed, after a proper choice of coordinates and constants, in the form

$$ds^2 = dt^2 - e^{at}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{9}$$

where a is the constant of integration. It may be observed that the model (9) becomes isotropic [19]. Also, the model (9) is expanding and has no singularity. The model reduces to a flat space time when $t = 0$.

3 Massive String Source

In recent years, there has been a lot of interest in cosmic strings and string cosmological models. Cosmic strings are one-dimensional topological defects associated with spontaneous symmetry breaking whose plausible production site is cosmological phase transitions in the early universe [8, 26]. The present-day configurations of the universe are not contradicted by large-scale network of strings in the early universe. The work of Reddy [18] gives a detailed review of string cosmologies investigated by several authors in general relativity and alternative theories of gravitation.

The energy momentum tensor for a cloud of massive strings can be written as [9, 25].

$$T_{ij} = \rho U_i U_j - \lambda X_i X_j \tag{10}$$

where ρ is the rest energy density of cloud of strings with particles attached to them, λ the tension density of strings, U^i the cloud four velocity and X^i is the string direction.

We have

$$U^i U_i = -X^i X_i = 1 \quad \text{and} \quad U^i X_i = 0 \tag{11}$$

We consider

$$\rho = \rho_p + \lambda$$

ρ_p being the particle energy density. The field equations (1) of bimetric theory for the metric (3) with the help of (4), (10) and (11) can be written as

$$\alpha_{44} = -32\pi\kappa\lambda \tag{12}$$

$$\alpha_{44} = 0 \tag{13}$$

$$3\alpha_{44} = 32\pi\kappa\rho \tag{14}$$

Equations (12)–(14) yields

$$\rho = \lambda = 0 \tag{15}$$

This immediately implies that spherically symmetric cosmic strings do not exist in Rosen's bimetric theory of gravitation. Hence in this case also, we get the vacuum model given by (9).

4 Scalar Meson Field

In this section we intend to construct cosmological model for attractive massive scalar meson field whose energy momentum tensor is given by

$$T_{ij} = V_i V_j - \frac{1}{2} g_{ij} \left(V_k V^k - m^2 V^2 \right) \tag{16}$$

together with

$$\sigma = g^{ij} V_{;ij} + m^2 V = 0 \tag{17}$$

where m is the mass parameter and σ is the source density of the scalar meson field V. Here afterwards the suffixes i and semicolon; after a field variable represent ordinary and covariant differentiations with respect to X^i and g_{ij} respectively.

The field equations (1) for the metrics (3) and (4) with the energy momentum tensor (16) can be written as

$$\alpha_{44} = -16\pi\kappa \left(V_4^2 - m^2 V^2 \right) \tag{18}$$

$$3\alpha_{44} = 16\pi\kappa \left(V_4^2 + m^2 V^2 \right) \tag{19}$$

Equations (18) and (19) yields two basic solutions for V as

$$V = A e^{\frac{m}{\sqrt{2}} t} \tag{20}$$

and

$$V = B e^{-\frac{m}{\sqrt{2}} t} \tag{21}$$

where A and B are initial values of V.

The source density of the scalar meson field given by (17) for the metric (3) can be obtained as

$$V_{44} + \frac{3}{2} \alpha_4 V_4 + m^2 V = 0 \tag{22}$$

Using the values of V from (20) and (21) we get

$$\alpha = -\sqrt{2}mt + C \tag{23}$$

and

$$\alpha = \sqrt{2}mt + D \tag{24}$$

where C and D are initial values of α.

The metrics corresponding to solutions (23) and (24) can be written as

$$ds^2 = dt^2 - e^{-\sqrt{2}mt}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{25}$$

and

$$ds^2 = dt^2 - e^{\sqrt{2}mt}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2\right) \tag{26}$$

respectively.

The model governed by (26) is an expanding model whereas the model in (25) represents a contracting model.

5 Conclusion

In this paper it is shown that in Rosen's bimetric theory of gravitation either the macro cosmological model representing perfect fluid or with massive string source do not survive whereas the micro cosmological model representing scalar meson field survives for spherically symmetric space time. However, at the early stages of evolution of the universe cosmic strings do appear which lead to formation of galaxies. Hence, it may be said that Rosen's bimetric theory of gravitation does not help to describe the early era of the universe.

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